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The ultimate theoretical error on γ from $B \rightarrow DK$ decays

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ABSTRACT: The angle γ of the standard CKM unitarity triangle can be determined from $B \rightarrow DK$ decays with a very small irreducible theoretical error, which is only due to second-order electroweak corrections. We study these contributions and estimate that their impact on the γ determination is to introduce a shift $|\delta\gamma| \lesssim \mathcal{O}(10^{-7})$, well below any present or planned future experiment.

KEYWORDS: Quark Masses and SM Parameters, B-Physics, CP violation, Standard Model

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1 Introduction

The determination of the standard CKM unitarity triangle angle $\gamma \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ from $B \rightarrow DK$ and $B \rightarrow \bar{D}K$ decays is theoretically extremely clean. The reason is that the $B \rightarrow DK$ transitions receive contributions only from tree operators, and none from penguin operators. Furthermore, all the relevant matrix elements can be obtained from data if enough D -decay channels are measured. The sensitivity to γ comes from the interference of $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ decay amplitudes, which have a relative weak phase γ , cf. figure 1. These quark-level transitions mediate $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$ decays, respectively. The D^0 and \bar{D}^0 subsequently decay into a common final state f , which allows the two decay channels to interfere. Several variants of the method have been proposed, distinguished by the final state f : i) f can be a CP eigenstate such as $K_S\pi^0$ and $K_S\phi$ [1, 2], ii) a flavor state such as $K^+\pi^-$ and $K^{*+}\rho^-$ [3, 4], or iii) a multibody state such as $K_S\pi^+\pi^-, \pi^+\pi^-\pi^0$ [5–7]. Other possibilities include the decays of neutral B mesons, B^0 and B_s , [8–13], multibody B decays [14–19] and D^* or D^{**} decays [20, 21] (see also the reviews in [22–24] and the current combination of LHCb measurements in [25]).

The above set of methods has several sources of theoretical errors. Most of them can be reduced once more statistics becomes available. For instance, in the past the $D \rightarrow K_S\pi^+\pi^-$ Dalitz plot needed to be modeled using a sum of Breit-Wigner resonances or using the K-matrix formalism. Utilizing the data from entangled $\psi(3770) \rightarrow D\bar{D}$ decays measured at CLEO-c [26] and BES-III, this uncertainty can in principle be completely avoided [6]. The related error is now statistics-dominated [27, 28].

Other sources of reducible uncertainties are $D - \bar{D}$ mixing and $K - \bar{K}$ mixing (for final states with K_S). Both of these effects can be included trivially by modifying the expressions for the decay amplitudes, taking meson mixing into account, and then using experimentally measured mixing parameters [29]. The effect of $D - \bar{D}$ mixing is most significant if the D decay information comes from entangled $\psi(3770) \rightarrow D\bar{D}$ decays. The shift in γ is then linear in x_D, y_D , giving $\Delta\gamma \leq 2.9^\circ$ [30] (see also [31]). For flavor-tagged D

decays (i.e. from $D^* \rightarrow D\pi$) the effect is quadratic in x_D, y_D and thus much smaller [32]. The effect of $K - \bar{K}$ mixing in D decay modes involving a neutral kaon was discussed recently in [33]; it introduces a shift in the extraction of γ of order 10^{-2} which can be systematically incorporated into the analysis. Similarly, for γ extraction from untagged $B_s \rightarrow D\phi$ decays the inclusion of $\Delta\Gamma_s$ can be important and can be achieved once $\Delta\Gamma_s$ is well measured [34].

In the extraction of γ from $B \rightarrow DK$, CP violation in the D system was usually neglected. Even if this assumption is relaxed, it is still possible to extract γ by appropriately modifying the expressions for the decay amplitudes (and using the fact that in Cabibbo-allowed D decays there is no direct CP violation¹) [25, 35–39].

Yet another source of reducible theory error are QED radiative corrections to the decay widths. The uncertainties from this source are expected to be below present experimental sensitivity on γ so that not much work has been done on them. Since the corrections are CP conserving they can be reabsorbed in the CP-even measured hadronic quantities and would not affect γ , as long as in the measurements the radiative corrections are treated consistently between different decay modes.

The first irreducible theory error on γ thus comes from higher-order electroweak corrections. This error cannot be eliminated using just experimental information and may well represent the ultimate precision of the γ determination from $B \rightarrow DK$ decays. The resulting uncertainty was estimated using scaling arguments in ref. [40] and found to be of the order of $\delta\gamma/\gamma \sim \mathcal{O}(10^{-6})$. In this paper we perform a more careful analysis, and find that the induced uncertainty is in fact most probably even an order of magnitude smaller. The one-loop electroweak corrections give rise to local and nonlocal contributions. We estimate the size of the local contributions using naive factorization and obtain $\delta\gamma/\gamma \lesssim \mathcal{O}(10^{-7})$. The nonlocal contributions are more difficult to estimate, but naively one expects that they are not significantly larger than the local ones.

The paper is organized as follows. In section 2 we give a brief discussion of electroweak corrections for $B \rightarrow DK$ decays with a focus on the γ extraction. We also give numerical estimates for the shift, $\delta\gamma$, utilizing the analytic results of section 3, where further details of the calculation are given. Finally, we conclude in section 4.

2 The shift in γ from $B \rightarrow DK$ due to electroweak corrections

The measurement of γ in $B \rightarrow DK$ decays is based on the interference between the tree-level $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ mediated processes, cf. figure 1. The sensitivity to the weak phase γ enters through the amplitude ratio

$$r_B e^{i(\delta_B - \gamma)} = \frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)}, \quad (2.1)$$

where $\delta_B = (114.8 \pm 9.4)^\circ$ is a strong phase, and $r_B = 0.0956 \pm 0.0063$ reflects the CKM and color suppression of the amplitude $A(B^- \rightarrow \bar{D}^0 K^-)$ relative to the amplitude $A(B^- \rightarrow D^0 K^-)$.

¹In fact, electroweak box diagrams give rise to tiny CP-violating effects which, for Cabibbo-allowed D decays, are of order $\lambda^{10} \cdot m_b^2/M_W^2 \approx 10^{-9}$, where $\lambda \equiv |V_{us}|$ — about two orders of magnitude smaller than the irreducible uncertainty we estimate below.



Figure 1. Tree contributions (with single W exchange) that mediate $b \rightarrow c\bar{u}s$ (left) and $b \rightarrow u\bar{c}s$ (right) quark-level processes, which lead to $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$ decays, respectively.

$D^0 K^-$) [41] (we use the latest (Winter 2012) update at <http://ckmfitter.in2p3.fr>). Here and below we focus on the charged $B^- \rightarrow DK^-$ and $B^- \rightarrow \bar{D}K^-$ decays. The results can be readily adapted also to other $B \rightarrow DK$ or $B_s \rightarrow D_s K$ decays used for extraction of γ .

The expression (2.1) is valid only at leading order in the weak interactions, $\mathcal{O}(G_F)$, when both the $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ transitions are mediated by the tree-level processes. At this order the two processes are described by the usual nonleptonic weak effective Hamiltonians

$$\mathcal{H}_{\bar{c}u}^{(0)} = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* [C_1(\mu) Q_1^{\bar{c}u} + C_2(\mu) Q_2^{\bar{c}u}], \quad (2.2)$$

$$\mathcal{H}_{\bar{u}c}^{(0)} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* [C_1(\mu) Q_1^{\bar{u}c} + C_2(\mu) Q_2^{\bar{u}c}], \quad (2.3)$$

where the four-fermion operators are

$$Q_1^{\bar{c}u} = (\bar{c}b)_{V-A} (\bar{s}u)_{V-A}, \quad Q_2^{\bar{c}u} = (\bar{s}b)_{V-A} (\bar{c}u)_{V-A}, \quad (2.4)$$

$$Q_1^{\bar{u}c} = (\bar{u}b)_{V-A} (\bar{s}c)_{V-A}, \quad Q_2^{\bar{u}c} = (\bar{s}b)_{V-A} (\bar{u}c)_{V-A}. \quad (2.5)$$

Above we have used the short-hand notation $(\bar{c}b)_{V-A} (\bar{s}u)_{V-A} \equiv (\bar{c}\gamma^\mu(1-\gamma_5)b) (\bar{s}\gamma_\mu(1-\gamma_5)u)$, and similarly for the other quark flavors. The scale at which the Wilson coefficients are evaluated is close to the b quark mass, $\mu \sim m_b$, with $C_1(m_b) = 1.10$, and $C_2(m_b) = -0.24$ at leading-log order [42], for $m_b(m_b) = 4.163$ GeV [43] and $\alpha_S(M_Z) = 0.1184$ [44]. The decay amplitudes in eq. (2.1) are then given at leading order in the electroweak expansion by

$$A(B^- \rightarrow \bar{D}^0 K^-) = \langle \bar{D}^0 K^- | \mathcal{H}_{\bar{u}c}^{(0)} | B^- \rangle, \quad \text{and} \quad A(B^- \rightarrow D^0 K^-) = \langle D^0 K^- | \mathcal{H}_{\bar{c}u}^{(0)} | B^- \rangle. \quad (2.6)$$

At second order in the weak interactions, $\mathcal{O}(G_F^2)$, there are corrections to (2.1) and (2.6) from W box diagrams, and from vertex corrections, shown in figure 2, and from double penguin diagrams. In addition there are also self-energy diagrams for the W -propagator and wave function renormalization diagrams for external legs, which however have exactly the same CKM structure as the leading order contributions and thus do not affect the γ extraction. The same is true of the vertex corrections due to a Z or W loop, shown in figure 2 (right), which correct the CKM matrix at one-loop. The double penguin insertions are two-loop and are thus subleading, as can be easily checked from the small sizes of the respective Wilson coefficients. They are safely neglected in the following.

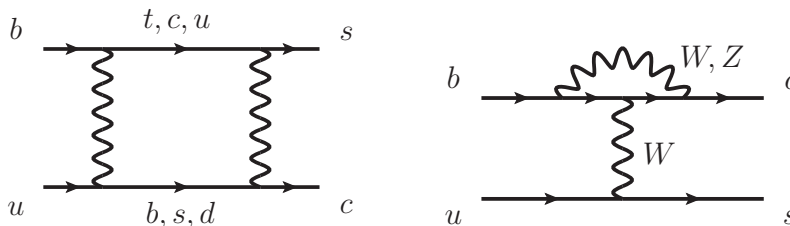


Figure 2. The electroweak corrections to $b \rightarrow c \bar{u} s$ process at order $\mathcal{O}(g^4)$, the box diagram (left) and vertex correction (right). Similar diagrams appear in $b \rightarrow u \bar{c} s$ processes.

The leading effect on extracted γ at $\mathcal{O}(G_F^2)$ then comes from the box diagram in figure 2 (left). The dominant contribution is effectively due to the top and bottom quark running in the loop, as we show in the next section. The CKM structure of the box diagram is different from that of the $\mathcal{O}(G_F)$ tree contribution and is given, for the $b \rightarrow c \bar{u} s$ transition, by

$$b \rightarrow c \bar{u} s : \text{ tree level } \sim V_{cb} V_{us}^*, \quad \text{ box diagram } \sim (V_{tb} V_{ts}^*)(V_{cb} V_{ub}^*). \quad (2.7)$$

Since the weak phases of the two contributions are different, this results in a shift $\delta\gamma$ in the extracted value of γ .

A similar higher-order electroweak diagram contributes also to the $b \rightarrow u \bar{c} s$ transition, which is given by exchanging the external u and c quarks in figure 2 (left). Again, the dominant contribution is effectively due to the top and bottom quark running in the loop, so that the CKM factors are

$$b \rightarrow u \bar{c} s : \text{ tree level } \sim V_{ub} V_{cs}^*, \quad \text{ box diagram } \sim (V_{tb} V_{ts}^*)(V_{ub} V_{cb}^*). \quad (2.8)$$

In this case the weak phases of the LO and NLO contributions are the same to a very good approximation, so that the electroweak contributions do not induce a shift in γ .

Keeping only the local part of the box diagram, the relevant change to the effective weak Hamiltonian is very simple. The structure of the CKM coefficients in (2.7) and (2.8) is such that all the corrections relevant for the γ extraction are in the $\mathcal{H}_{\bar{c}u}$ effective weak Hamiltonian eq. (2.2), which at $\mathcal{O}(G_F^2)$ takes the form

$$\mathcal{H}_{\bar{c}u}^{(1)} = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* [(C_1(\mu) + \Delta C_1(\mu)) Q_1^{\bar{c}u} + (C_2(\mu) + \Delta C_2(\mu)) Q_2^{\bar{c}u}]. \quad (2.9)$$

The Wilson coefficients $C_{1,2}(\mu)$ are the same Wilson coefficients as in eqs. (2.2) and (2.3), while $\Delta C_{1,2}(\mu)$ are calculable corrections. They depend on the CKM elements and carry a weak phase γ . They therefore have a different weak phase than $C_{1,2}(\mu)$, which in our phase convention are real. This introduces a shift in $\delta\gamma$ in the extraction of the weak phase γ from $B \rightarrow DK$ decays. This shift represent the ultimate theory error on the measurement of γ .

Defining the ratio of matrix elements for the two relevant operators

$$r_A \equiv \frac{\langle K^- D^0 | Q_2^{\bar{c}u} | B^- \rangle}{\langle K^- D^0 | Q_1^{\bar{c}u} | B^- \rangle}, \quad (2.10)$$

the shift in the ratio r_B , eq. (2.1), is

$$r_B e^{i(\delta_B - \gamma)} \rightarrow r_B e^{i(\delta_B - \gamma)} \left(1 - \frac{\Delta C_1}{C_1 + C_2 r_A} - \frac{\Delta C_2}{C_1/r_A + C_2} \right), \quad (2.11)$$

where we expanded in the small corrections ΔC_1 , ΔC_2 to linear order. The resulting shift in the extracted value of γ is

$$\delta\gamma = \frac{\text{Im}(\Delta C_1)}{C_1 + C_2 r_A} + \frac{\text{Im}(\Delta C_2)}{C_1/r_A + C_2}. \quad (2.12)$$

The size of the corrections $\Delta C_{1,2}$ will be calculated in the next section, while here we only quote the numerical results. The unresummed result for $\text{Im}(\Delta C_2)$, cf. eq. (3.5) below, is

$$\text{Im}(\Delta C_1) = 0, \quad \text{Im}(\Delta C_2) = (5.3 \pm 0.3) \cdot 10^{-8} \times \sin \gamma, \quad (2.13)$$

where the error only reflects the experimental errors due to the input parameters. The results with $\log(m_b/M_W)$ resummed, cf. eq. (3.22) below, are

$$\text{Im}(\Delta C_1) = (4.5 \pm 0.2) \cdot 10^{-9} \times \sin \gamma, \quad \text{Im}(\Delta C_2) = (4.3 \pm 0.2) \cdot 10^{-8} \times \sin \gamma. \quad (2.14)$$

In order to obtain $\delta\gamma$ we also need to estimate the ratio of the matrix elements, r_A , in (2.10). In naive factorization this ratio is

$$r_A = \frac{f_D F_0^{B \rightarrow K}(0)}{f_K F_0^{B \rightarrow D}(0)} = 0.4, \quad (2.15)$$

where we used $f_D = 0.214 \text{ GeV}$ [45], $F_0^{B \rightarrow K}(0) = 0.34$ [46], $f_K = 0.16 \text{ GeV}$, $F_0^{B \rightarrow D}(0) = 1.12$ [47]. In eq. (2.15) we only quote the central value, since the error on this estimate is bigger than the errors on the form factors themselves. However, we do not expect the error on the estimate of r_A in (2.15) to be bigger than a factor of a few.

Using this and setting $\gamma = 68^\circ$ for definiteness, we obtain the estimate for the shift $\delta\gamma$,

$$\delta\gamma \simeq 2.0 \cdot 10^{-8} \quad (2.16)$$

where to this accuracy the resummed expressions for $\Delta C_{1,2}$ (with nonlocal contributions neglected) and unresummed results coincide. An uncertainty of at most an additional factor of a few can be expected on the above estimate, so that we can conclude that the ultimate theoretical error on γ measurement is safely below

$$|\delta\gamma| \lesssim 10^{-7}. \quad (2.17)$$

In the next section we derive the analytic expressions for $\Delta C_{1,2}(\mu)$, and then draw our conclusions in section 4.

3 Corrections to the electroweak Hamiltonian

In this section we consider the $b \rightarrow c\bar{u}s$ box diagram, figure 2 (left), in detail. The results can be readily adapted to the $b \rightarrow u\bar{c}s$ case by exchanging the external quarks and adjusting the CKM factors. The diagram in figure 2 (left) is superficially similar to the box diagrams contributing to $\bar{K}^0 - K^0$ and $\bar{B}_{(s)}^0 - B_{(s)}^0$ mixing [42], and to $b \rightarrow ss\bar{d}, dd\bar{s}$ decays [48]. The difference is that the box diagram in figure 2 (left) has both up- and down-quarks running in the loop, in contrast to the case of $\bar{K}^0 - K^0$ and $\bar{B}_{(s)}^0 - B_{(s)}^0$ mixing where both quarks in the loop are of up-type.

We will calculate the shift $\delta\gamma$ in two ways — first by keeping only the $\log(m_b/M_W)$ enhanced local contribution, but without resumming it. Subsequently we will resum this log. In the first case we will take b , t and W in the loop to be heavy and integrate them out at $\mu \sim M_W$. In this way one obtains the local operator part of the effective field theory (EFT) with only the light quarks, u, d, s, c , and an external non-dynamical b -quark field. Keeping only the local operators in EFT is a crude approximation that does, however, suffice for our purposes — to show that the induced corrections on the γ extraction are exceedingly small. The obtained result will also give us better understanding of the correct EFT results with resummed $\log(m_b/M_W)$, which we will perform next. The resummation is achieved by first integrating out t and W at $\mu \sim M_W$ and matching onto the effective theory with b , and c, s, d, u quarks. We will then evolve the Wilson coefficients down to the scale $\mu \sim m_b$ using the renormalization-group (RG).

3.1 The result without resummations

We first evaluate the box diagram at $\mu \sim M_W$, treating t and b quarks as massive and u, c and d, s quarks as massless, and set all external momenta to zero (including the external b -quark momentum). This will give us the local part of the EFT contributions with unre-summed Wilson coefficients. Because of the double GIM mechanism, acting on both the internal up-quark and down-quark lines, the leading contribution is proportional to $x_t y_b$, where $x_t \equiv m_t^2/M_W^2$, $y_b \equiv m_b^2/M_W^2$. This is easy to see by expanding the matrix element for the box-diagram correction to the $B \rightarrow DK$ decay in terms of the quark masses,

$$A_{\text{box}} = \sum_{u_i=u,c,t} \sum_{d_j=d,s,b} \frac{G_F^2}{2} \lambda_{u_i}^{b \rightarrow s} \lambda_{d_j}^{u \rightarrow c} \left\{ A_1 M_W^2 + A_2 m_{d_j}^2 + A_3 m_{u_i}^2 + \right. \\ \left. + A_4 m_{u_i}^2 m_{d_j}^2 + \dots \right\} \times \langle DK^- | (\bar{s}b)_{V-A} (\bar{c}u)_{V-A} | B^- \rangle. \quad (3.1)$$

The CKM factors $\lambda_{d_j}^{u \rightarrow c} = V_{ud_j}^* V_{cd_j}$ and $\lambda_{u_i}^{b \rightarrow s} = V_{u_i b} V_{u_i s}^*$ are associated with the flavor transitions on the internal down- and up-quark lines in figure 2 (left), respectively. The contributions in the first line, proportional to M_W^2 , $m_{d_j}^2$ and $m_{u_i}^2$, vanish because either $\sum_{u_i=u,c,t} \lambda_{u_i}^{b \rightarrow s} = 0$ or $\sum_{d_j=d,s,b} \lambda_{d_j}^{u \rightarrow c} = 0$.

Ignoring nonlocal contributions (see below), the box diagram with b and t quark massive and all the other quarks massless therefore matches onto the effective Hamiltonian (2.9). This amounts to a matching calculation where t and b quarks are integrated out simultaneously at $\mu \sim M_W$ and results in a change ΔC_2 of the Wilson coefficient C_2 in eq. (2.9), given by

$$\Delta C_2 = \frac{\alpha}{4\pi \sin^2 \theta_w} \frac{V_{tb} V_{ts}^* V_{ub}^*}{V_{us}^*} \hat{C}(x_t, y_b) = - \left| \frac{\alpha}{4\pi \sin^2 \theta_w} \frac{V_{tb} V_{ts} V_{ub}}{V_{us}} \right| \hat{C}(x_t, y_b) e^{i\gamma}. \quad (3.2)$$

The Wilson coefficient C_2 in (2.3) receives a similar correction but with the same weak phase as the $\mathcal{O}(G_F)$ term. Thus the correction does not contribute to $\delta\gamma$ and we neglect it, cf. eqs. (2.7), (2.8). The result of our calculation agrees with the result extracted from [49]

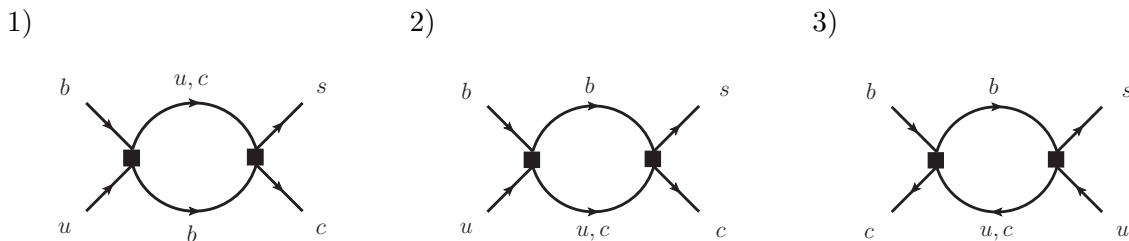


Figure 3. The double insertion $T\{Q_1, Q_1\}$, diagram 1), and $T\{Q_2, Q_2\}$, diagrams 2) and 3), contributing to the mixing into \tilde{Q}_2 .

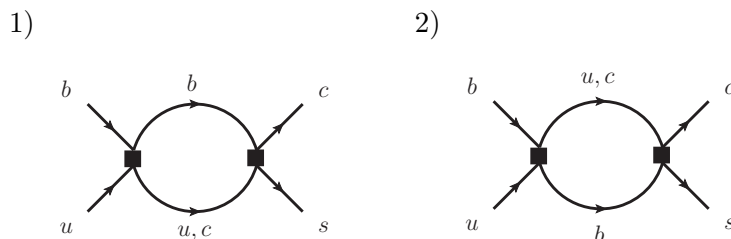


Figure 4. The double insertions $T\{Q_1, Q_2\}$ contributing to the mixing into the operator \tilde{Q}_1 .

and reads

$$\hat{C}_{\text{full}}(x_t, y_b) = -\frac{x_t y_b}{8} \left[\frac{9}{(x_t - 1)(y_b - 1)} + \left(\frac{(x_t - 4)^2}{(x_t - 1)^2(x_t - y_b)} \log x_t + (x_t \leftrightarrow y_b) \right) \right]. \quad (3.3)$$

Note that the loop function $\hat{C}_{\text{full}}(x, y)$ vanishes if either $x \rightarrow 0$ or $y \rightarrow 0$. This proves that the only nonzero contribution in (3.1) is $A_4 \propto x_t y_b$. In fact, it is a very good approximation to keep in this result only the $\log y_b$ enhanced contribution,

$$\hat{C}(x_t, y_b) = 2y_b \log y_b + \mathcal{O}(y_b), \quad (3.4)$$

where the finite terms amount to an $\mathcal{O}(10\%)$ correction. Using the values for the CKM matrix elements from the CKMfitter collaboration [41] and further input from [44], we find

$$\Delta C_2 = (5.3 \pm 0.3) \cdot 10^{-8} \times e^{i\gamma}, \quad (3.5)$$

where the error shown is only due to the CKM elements.

The Wilson coefficient $\hat{C}(x_t, y_b)$ contains the unresummed large logarithm $\log y_b$. The logarithm is multiplied by $2y_b$ and would vanish in the limit of zero b quark masses. However, since the Wilson coefficient $\hat{C}(x, y)$ starts only at $\mathcal{O}(y_b)$, the term with $\log y_b$ represents a large correction. In the next subsection we therefore perform a resummation of this logarithm.

3.2 The resummed result

In order to resum $\log(m_b/M_W)$ we need to explicitly keep the hierarchy of scales, $m_b \ll M_W$, in the construction of the effective theories. For $\mu > M_W$ one has the full SM, for

$m_b < \mu < M_W$ one has an effective theory with massless b and c, s, d, u quarks but no top quark, while below m_b there is an effective theory with only the light quarks, c, s, d, u .

In the matching at $\mu \sim M_W$ the top quark and the W, Z bosons are integrated out, while the massless bottom quark is still a dynamical degree of freedom also in the effective theory — this is the main difference to the previous subsection. Integrating out the W at tree level in electroweak counting generates the effective Hamiltonians (2.2), (2.3) and its variants containing also the dynamical d -quark field. The contribution proportional to y_b now vanishes at the electroweak scale to the order considered. However, this contribution will be generated by mixing of two insertions of dimension-six operators below the electroweak scale. It is therefore useful to write the Hamiltonian describing the five-flavor effective theory in the following way,

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{f=5} = & \frac{G_F}{\sqrt{2}} \sum_{\substack{u_{1,2}=u,c \\ d_{1,2}=s,d,b}} V_{u_1 d_2} V_{u_2 d_1}^* \sum_{i,j=1}^2 C_i(\mu) Z_{ij} Q_j^{(u_1 d_2; d_1 u_2)} \\ & + 2G_F^2 V_{cb} V_{us}^* \cdot \left| \frac{V_{tb} V_{ts} V_{ub}}{V_{us}} \right| e^{i\gamma} \left[\sum_{i,j,k=1}^2 C_i C_j \hat{Z}_{ij,k} \tilde{Q}_k + \sum_{l,k=1}^2 \tilde{C}_l \tilde{Z}_{lk} \tilde{Q}_k \right], \end{aligned} \quad (3.6)$$

where we used $V_{tb} V_{ts}^* = -V_{cb} V_{cs}^* + \mathcal{O}(\lambda^2)$, with $\lambda = |V_{us}| \simeq 0.23$ (numerically, this replacement is valid up to a three-permil correction). Moreover, we denoted the usual four-quark operators by

$$Q_1^{(u_1 d_2; d_1 u_2)} = (\bar{u}_1 d_2)_{V-A} (\bar{d}_1 u_2)_{V-A}, \quad Q_2^{(u_1 u_2; d_1 d_2)} = (\bar{u}_1 u_2)_{V-A} (\bar{d}_1 d_2)_{V-A}, \quad (3.7)$$

and defined

$$\tilde{Q}_1 = \frac{m_b^2}{\mu^{2\epsilon} g_s^2} (\bar{s} u)_{V-A} (\bar{c} b)_{V-A}, \quad \tilde{Q}_2 = \frac{m_b^2}{\mu^{2\epsilon} g_s^2} (\bar{s} b)_{V-A} (\bar{c} u)_{V-A}. \quad (3.8)$$

The last two operators denoted by a tilde are formally of dimension eight because of the m_b^2 factor. They have the same four-quark structure as the leading power operators $Q_{1,2}$ so that their contributions could be absorbed by redefining the Wilson coefficients $C_{1,2}$ allowing them to be complex. It is more practical, however, to keep the Wilson coefficients real and split-off explicitly the contributions to the effective Hamiltonian that carry the extra weak phase as we did in (3.6). Note that in the second line in eq. (3.6) we neglect all the $\mathcal{O}(G_F^2)$ terms with the same weak phase as the $\mathcal{O}(G_F)$ terms in eqs. (2.2), (2.3), since these are not relevant for calculating $\delta\gamma$. We also neglect the six-quark operators which arise from integrating out the W boson and the top quark, as they are suppressed by an additional factor of $1/M_W^2$.

The dimension-eight Wilson coefficients at the electroweak scale vanish to leading order. The mixing of double insertions of dimension-six operators into $\tilde{Q}_{1,2}$ will generate non-vanishing Wilson coefficients $\tilde{C}_{1,2}(\mu)$ below the electroweak scale. The inverse powers of g_s in the definition of $\tilde{Q}_{1,2}$ in (3.8) take into account that we will sum the leading logarithms proportional to the strong coupling constant.

Let us now look at some of the contributing terms in more detail. The sum of the two diagrams denoted by 2) in figure 3 yields

$$\begin{aligned}
 & C_2^2 V_{cb} V_{us}^* (|V_{ub}|^2 + V_{cs}^* V_{cb} \frac{V_{ub}^*}{V_{us}^*}) \langle Q_2 Q_2 \rangle_{\text{div}} = C_2^2 V_{cb} V_{us}^* (|V_{ub}|^2 + |V_{cb}|^2 \frac{V_{cs}^* V_{ub}^*}{V_{cb}^* V_{us}^*}) \langle Q_2 Q_2 \rangle_{\text{div}} \\
 & = C_2^2 V_{cb} V_{us}^* (|V_{ub}|^2 + |V_{cb}|^2 \left| \frac{V_{cs} V_{ub}}{V_{cb} V_{us}} \right| e^{i\gamma}) \langle Q_2 Q_2 \rangle_{\text{div}} = C_2^2 V_{cb} V_{us}^* \left| \frac{V_{cb} V_{cs} V_{ub}}{V_{us}} \right| e^{i\gamma} \langle Q_2 Q_2 \rangle_{\text{div}} + \dots,
 \end{aligned} \tag{3.9}$$

where $\langle Q_2 Q_2 \rangle_{\text{div}}$ is the common divergence of the two diagrams, which is independent of the light-quark masses. In the last step we kept only the term proportional to the factor with a weak phase, which is the only contribution entering the shift $\delta\gamma$. The Lorentz and color structure of $\langle Q_2 Q_2 \rangle_{\text{div}}$ is the same as of \tilde{Q}_2 , so that this gives the anomalous dimension of the double insertion mixing into \tilde{Q}_2 . The sum of the two diagrams denoted by 1) in figure 3 is similar to the first case, eq. (3.9), but with the replacement $C_2 \rightarrow C_1$, $Q_2 \rightarrow Q_1$. The sum of the two diagrams denoted by 3) in figure 3 yields

$$C_2^2 V_{cb} V_{us}^* (|V_{ub}|^2 + |V_{cb}|^2) \langle Q_2 Q_2 \rangle_{\text{div}}, \tag{3.10}$$

and does not carry a weak phase. As such it does not contribute to $\delta\gamma$ and can be discarded. There are also four additional diagrams, shown in figure 4, which lead to the mixing of double insertions into the Fierz-transformed operator \tilde{Q}_1 .

To obtain the contributions of double $\mathcal{H}_{\text{eff}}^{f=5}$ insertions to the running of $\tilde{Q}_{1,2}$ we thus only need to compute the diagrams denoted by 1) and 2) in figure 3, with a double insertion of Q_1 and Q_2 , respectively, plus two additional diagrams with an insertion of Q_1 and then Q_2 at each of the two weak vertices, cf. figure 4. We expand $\hat{\gamma}_{i,j;k} = \frac{\alpha_s}{4\pi} \hat{\gamma}_{i,j;k}^{(0)} + \dots$, where i, j denote the $Q_{1,2}$ insertions, and k is the labeling of the \tilde{Q}_k operators. Extracting $\hat{\gamma}_{i,j;k}^{(0)}$ from the one-loop divergence of the double insertion (see, for instance, [50] for details), our calculation yields

$$\hat{\gamma}_{1,1;2}^{(0)} = \hat{\gamma}_{2,2;2}^{(0)} = -8, \quad \hat{\gamma}_{1,2;1}^{(0)} = \hat{\gamma}_{2,1;1}^{(0)} = 8, \tag{3.11}$$

with all the remaining entries either vanishing or not contributing. The initial conditions for the dimension-six Wilson coefficients are given by $C_1(\mu_W) = 1$, $C_2(\mu_W) = 0$ to leading order [42]. Expanding $\tilde{C}_k = \tilde{C}_k^{(0)} + \mathcal{O}(\alpha_s)$, we find $\tilde{C}_k^{(0)}(\mu_W) = 0$ at leading order. A nonvanishing value will be induced by RG running for $\mu < \mu_W$, which we compute by solving

$$\mu \frac{d}{d\mu} \tilde{C}_k = \sum_l \tilde{C}_l \gamma_{lk} + \sum_{ij} C_i C_j \hat{\gamma}_{ij,k}, \tag{3.12}$$

where γ_{lk} is the well-known anomalous dimension for the mixing of the $Q_{1,2}$ operators,

$$\gamma_{lk} = \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix}. \tag{3.13}$$

It is advantageous to go to the diagonal basis of the current-current operators, by defining

$$Q_{\pm} = \frac{1}{2}(Q_1 \pm Q_2), \quad \tilde{Q}_{\pm} = \frac{1}{2}(\tilde{Q}_1 \pm \tilde{Q}_2). \quad (3.14)$$

In this way eq. (3.12) gets rewritten as a homogeneous equation [51], for which the standard techniques of obtaining closed expressions for the RG evolution apply. The transformed LO anomalous dimensions and the Wilson coefficients are [50]

$$\gamma'^{(0)} = R\gamma^{(0)}R^{-1}, \quad \hat{\gamma}'_{ij;k}{}^{(0)} = R_{im}R_{jn}\hat{\gamma}_{mn;l}^{(0)}R_{lk}^{-1}, \quad C'^{(0)} = (R^{-1})^T C^{(0)}, \quad (3.15)$$

where

$$R = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (3.16)$$

By explicit calculation we find

$$\hat{\gamma}'_{+,+;-}{}^{(0)} = 8, \quad \hat{\gamma}'_{-,-;+}{}^{(0)} = -8, \quad (3.17)$$

while the remaining entries are zero. Defining $D_+ \equiv (C_-, \tilde{C}_+/C_-)^T$, the renormalization-group equations for \tilde{C}_+ and C_- can be combined into

$$\mu \frac{dD_+}{d\mu} = \gamma_{D_+} \cdot D_+, \quad \text{where} \quad \gamma_{D_+} = \begin{pmatrix} \gamma_- & 0 \\ \hat{\gamma}_{-,-;+} & \gamma_+ - \gamma_- + 2\gamma_m - 2\beta \end{pmatrix}. \quad (3.18)$$

Here $\gamma_+ = 4$ and $\gamma_- = -8$ are the eigenvalues of the matrix (3.13). We obtain the corresponding solution for \tilde{C}_- and C_+ by exchanging the subscripts $+\leftrightarrow -$. Note that we have also included the running of the mass and the coupling constant related to the factor m_b^2/g_s^2 in the definition of the operators \tilde{Q}_k , given by the anomalous dimension of the quark mass γ_m and the QCD beta function β . Transforming back to the original basis, we find numerically

$$\{\tilde{C}_1(m_b), \tilde{C}_2(m_b)\} = \{0.03, 0.31\}, \quad (3.19)$$

where we used $\alpha_s(M_Z) = 0.1184$ [44] and $m_b(m_b) = 4.163 \text{ GeV}$ [43]. Note that the RG running has now also induced a nonzero correction to C_1 in (2.9), in contrast to the unresummed result. We used the mathematica package “RunDec” [52] for the numerical running of the strong coupling constant.

Finally, at the bottom-quark scale we need to calculate the $B \rightarrow DK$ matrix elements using our EFT Hamiltonian (3.6) in order to obtain the shift $\delta\gamma$. This will give the leading y_b behavior with resummed logarithms. We write the matrix elements suggestively as

$$\begin{aligned} \sum_k \Delta C_k(\mu_b) \langle Q_k \rangle(\mu_b) &= 2\sqrt{2}G_F \left| \frac{V_{tb}V_{ts}V_{ub}}{V_{us}} \right| e^{i\gamma} \left[\sum_{i,j=1}^2 C_i(\mu_b) C_j(\mu_b) \langle Q_i Q_j \rangle(\mu_b) \right. \\ &\quad \left. + \sum_{i=1,2} \tilde{C}_i(\mu_b) \langle \tilde{Q}_i \rangle(\mu_b) \right]. \end{aligned} \quad (3.20)$$

Here we expand $\Delta C_k = \frac{4\pi}{\alpha_s} \Delta C_k^{(0)} + \mathcal{O}(1)$; note that in this way the artificially inserted factor of $1/g_s^2$ in the definition of \tilde{Q}_k (3.8) is canceled. At LO it is not necessary to compute the double insertions $\langle Q_i Q_j \rangle$ since these are loop suppressed, and therefore we effectively obtain the matching condition for the Wilson coefficients of the local operators (2.9)

$$\Delta C_k^{(0)}(\mu_b) = 2m_b^2 \frac{\sqrt{2}G_F}{16\pi^2} \left| \frac{V_{tb}V_{ts}V_{ub}}{V_{us}} \right| e^{i\gamma} \tilde{C}_k^{(0)}(\mu_b). \quad (3.21)$$

Numerically, we find

$$|\Delta C_1| = (4.5 \pm 0.2) \cdot 10^{-9}, \quad |\Delta C_2| = (4.3 \pm 0.2) \cdot 10^{-8}; \quad (3.22)$$

the errors reflect the uncertainty in the electroweak input parameters. This should be compared to the unresummed result eq. (3.5). Expanding the solution of the renormalization-group equations around $\mu = M_W$ and expressing G_F in terms of the weak mixing angle we recover exactly the logarithm in eq. (3.4):

$$\Delta C_1 = 0, \quad \Delta C_2 = 2y_b \frac{\alpha}{16\pi \sin^2 \theta_w} (-4 \log y_b). \quad (3.23)$$

4 Conclusions

The determination of the SM weak phase γ from the $B \rightarrow DK$ decays has a very small irreducible theoretical error which is due to one-loop electroweak corrections. In this paper we have estimated the resulting shift in γ . Treating $m_b \sim M_W$ or resumming logs of m_b/M_W gives in both cases an estimated shift $\delta\gamma \sim 2 \cdot 10^{-8}$, keeping only the local operator contributions at the scale $\mu \sim m_b$. It is unlikely that the neglected non-local contributions, which come with the same CKM suppression as the local contributions, would differ from the above estimate by more than a factor of a few. For instance, in $K^0 - \bar{K}^0$ mixing the long distance contributions to $\text{Im}M_{12}$ are even an order of magnitude smaller than the short distance ones [53]. We can thus safely conclude that the irreducible theoretical error on the extraction of γ from $B \rightarrow DK$ is $|\delta\gamma| \lesssim \mathcal{O}(10^{-7})$.

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